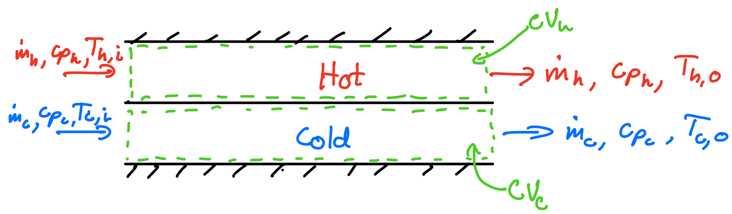
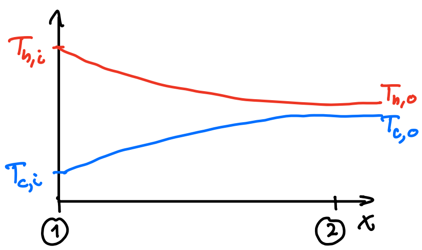
12A - 2ND CLASS

INTRO TO $\underbrace{\text{HEAT EXCHANGERS}}_{\text{"HX"}}$

$T_{h,o}$ — hot/cold — inlet/outlet

Parallel Flow HX





$\underbrace{T_{c,o} < T_{h,o}}$ not always true, depends on heat exchanger type

Hot Fluid

$$\cancel{\dot{E}_{st}}^{\,0} = \dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_g}^{\,0}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}_h cp_h T_{h,i} = \dot{m}_h cp_h T_{h,o} + q$$

$$q = \dot{m}_h cp_h (T_{h,i} - T_{h,o})$$

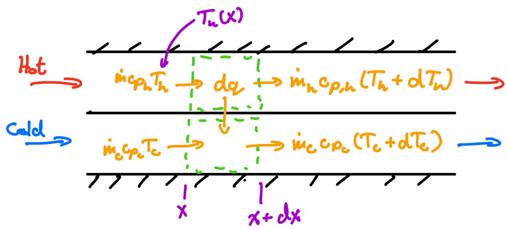
$C_h \equiv \dot{m}_h cp_h$ = Heat Capacity Rate [J/ks]

Cold Fluid

$$q = C_c (T_{c,o} - T_{c,i})$$

$$\boxed{q = C_h (T_{h,i} - T_{h,o})}$$

$$\boxed{q = C_c (T_{c,o} - T_{c,i})}$$



Energy Balance:

Hot Fluid:

$$\boxed{dq = -C_h\, dT_h}$$

Cold Fluid:

$$\boxed{dq = C_c\, dT_c}$$

Recall,

$$dq = U\, dA\, \Delta T$$

U: overall heat transfer coefficient [W/m²K]

dA: differential area

$\Delta T = T_h(x) - T_c(x)$

Note,

$\Delta T = T_h(x) - T_c(x)$

$$d(\Delta T) = dT_h - dT_c$$

$$= \frac{-dq}{C_h} - \frac{dq}{C_c}$$

$$d(\Delta T) = -dq\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\boxed{dq = \frac{-d(\Delta T)}{\frac{1}{C_h} + \frac{1}{C_c}}}$$

Relates dq & $d(\Delta T)$

So,

$$\frac{-d(\Delta T)}{\frac{1}{C_h} + \frac{1}{C_c}} = U\, dA\, \Delta T \Rightarrow \int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)\int_1^2 dA$$

from graph on first page

$$\Rightarrow \ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\frac{1}{C_h} = \frac{T_{h,i} - T_{h,o}}{q}$$

$$\frac{1}{C_c} = \frac{T_{c,o} - T_{c,i}}{q}$$

Rearrange,

$$\boxed{q = UA\, \frac{(\Delta T_2 - \Delta T_1)}{\ln(\Delta T_2 / \Delta T_1)}} = UA\, \Delta T_{lm}$$

log mean tempeture difference

$$\approx \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$